Fractal-like tree networks reducing the thermal conductivity

Boming Yu[1,*] and Baowen Li[1,2]

[1]*Department of Physics and Center for Computational Science and Engineering, National University of Singapore, Singapore 117542*

[2]*Laboratory of Modern Acoustics and Institute of Acoustics, Nanjing University, 210093, People's Republic of China*

(Received 3 November 2005; published 6 June 2006)

The effective thermal conductivity of composites with embedded self-similar H-shaped fractal-like tree networks is studied. It is found that the effective thermal conductivity of the composites with these networks is related to the structures of the networks and the ratio of the component thermal conductivities: the longer the branches, the lower the thermal conductivity; the smaller the ratio (β) of successive branch diameters, the lower the thermal conductivity; the denser the network, the lower the thermal conductivity. It is also found that the thermal conductivity of the H-shaped fractal-like tree networks does not obey Murray's law. The present results show that a network embedded in a composite plays an important role, and the thermal conductivity of the network itself may be less than that of the original material by several orders of magnitude. Fractal-like tree networks can significantly reduce the thermal conductivity compared to an equivalent single cylinder.

DOI: 10.1103/PhysRevE.73.066302 PACS number(s): 44.30.+v, 44.35.+c, 89.75.Da

I. INTRODUCTION

Tree networks have received increasing attention because of their wide existence in nature, as in lungs, vasculatures, botanical trees, and river basins, and their relevance to many real systems and applications such as the worldwide web, the internet, and social and energy transport networks. It has been shown that in natural systems tree networks often give the minimal resistance and optimal vascular diameter for driving the blood in mammals and water in plants. These mechanisms have recently been applied in design of energy transport systems and cooling systems of electronic chips due to increasing miniaturization of chips in microelectronic equipment and the production of redundant heat.

Bejan [1–3] developed a solution to the fundamental problem of how to collect the heat generated volumetrically in a low-conductivity volume and then how to transfer the heat to one point (heat sink). The solution was obtained as a sequence of optimization and organization steps. The sequence has a definite time direction; it begins with the smallest building block (element system) and proceeds toward larger building blocks (assemblies), and then finds the optimized shape and size of the assembly. It was shown that in an optimal design the high-conductivity material forms a treelike network, and this structure has the minimum thermal resistance and minimum entropy generation if the total heat flow rate is fixed and the volume fraction of high-conductivity material is also fixed. Bejan calls this theory a constructal theory. Neagu and Bejan [4] showed that the global thermal resistance between a volume and one point can be reduced to unprecedented levels by shaping the external boundary of each volume element. The volume is covered in a sequence of optimization and assembly steps that proceeds toward larger sizes. The resulting architecture is a leaflike tree structure with high-conductivity nerves and low-conductivity leaves. The fractal-like character of these designs and their relevance to the trend toward fractal-like properties in natural flow structures are discussed. The authors later extended their analysis to three-dimensional structures [5]. The geometry of each volume element, and the shape and distribution of high-conductivity inserts are optimized. The optimized architecture is pineconelike, with high-conductivity nerves and low-conductivity filling and heat-generating material. Recently, Gosselin and Bejan [6] also considered the problem of cooling a two-dimensional heat generating conducting volume with one heat sink, when the internal structure is so small that the conventional description of conduction breaks down. The effective thermal conductivity exhibits the "size effect," and is governed by the smallest structural dimension, which is comparable with the mean free path of the energy carriers. According to the constructal method, this starts at the elemental level, where there is only one high-conductivity layer for collecting and evacuating the heat. The shape of the smallest volume can be optimized for minimal thermal resistance. The construction reveals an internal multiscale structure shaped like a tree, where the spaces between the smallest branches are ruled by nanoscale heat transfer. It was shown that the transition from regions with nanoscale heat transfer to regions with conventional heat transfer is governed not only by the smallest dimensions, but also by heterogeneity (relative amounts of high and low conductivity). This constructal method and principle have also been used to analyze the convective heat transfer and pressure drop in fractal-like tree networks [7–11]. The results indicate that these structures can increase total convective heat transfer rate and offer the minimal resistance to flow and minimal pressure drop in a fluid, compared with the conventional parallel channels.

Usually, insulation materials such as foam and sponge cannot sustain high mechanical strength although they have good performance for heat insulation. However, if a treelike branching structure consisting of a material of high mechanical strength is placed in such an insulation material, a composite with high mechanical strength is formed. It is ex-

*Permanent address. Department of Physics, Huazhong University of Science and Technology, Wuhan, 430074, People's Republic of China. Email address: yuboming2003@yahoo.com.cn

pected that its properties (mechanical, thermal, and electrical) might be different from those of its original components.

In this paper we study the thermal conductivity of a composite with an embedded self-similar H-shaped fractal-like tree structure or network based on the constructal theory and the thermal-electrical analogy technique. The results might find applications in design of insulating or conducting composites and structures.

II. THERMAL CONDUCTIVITY OF COMPOSITES WITH EMBEDDED FRACTAL-LIKE TREE NETWORKS

Bejan *et al.* [12] applied the method of thermodynamic optimization to several classes of simple flow systems consisting of T- and Y-shaped assemblies of ducts, channels, and streams. In each case, the objective was to identify the geometric configuration that maximized performance subject to several global constraints. For simplicity in this analysis, we start with the H-shaped structure, a possible smallest structure. The H-shaped structure can be considered as a doubled T-shaped structure. The self-similar H-shaped fractal-like tree network structure of different branching levels is shown in Fig. 1; it can be built by repeating a finite number of elements shaped as H and by assuming a fixed area (volume) and length ratio. This results in an increasing number of channels with slender branches. If the volume of all branches is neglected or stems are assumed to be infinitely thin, the structures as shown in Fig. 1 are fractals [13]. Figure 1(a) is the structure of fractal dimension $D=2$ and of branching angle π yielding a plane-filling tree [13]; Fig. 1(b) shows a typical branch of a half H due to symmetry at some intermediate level k, i.e., the kth level of the network. Figure 1(c) is a structure of fractal dimension $D=1.26$, much below 2, producing a sparsely plane-filling tree [13]. Since the present work deals with the self-similar tree networks with finite volume branches, there is a possible smallest element or unit structure (an H structure) because the daughter branches may touch mother stems after finite repeats. Therefore, the structures in this paper are called fractal-like tree networks.

In Fig. 1(b), d_k and L_k are the branch diameter and branch length of the kth level, respectively, and d_{k+1} and L_{k+1} are, respectively, the branch diameter and branch length of the $(k+1)$th level. Each tube or cylinder branches into n (=2 in Fig. 1) smaller ones.

We assume that each branch of the network is a smooth cylinder or channel, which is composed of a material of high thermal conductivity λ, and of high mechanical strength, such that this material also serves as the supporting material in a matrix material and thus forms a composite. We also assume that the left and right boundaries of this network placed in a two-dimensional medium of low thermal conductivity are kept adiabatic, and the upper and lower boundaries are, respectively, kept at higher temperature T_H and lower temperature T_L (see Fig. 1), so the one-dimensional heat flow model is applicable. This one-dimensional heat flow model was used to calculate the effective thermal conductivity of heterogeneous media based on the thermal-electrical analogy by many others [14–18].

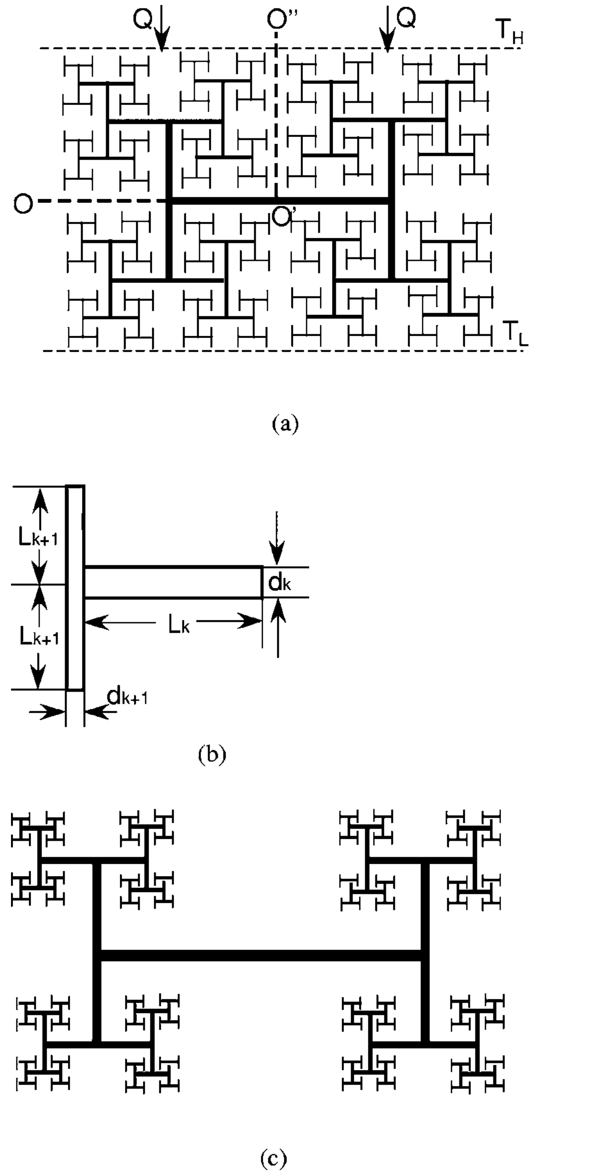

FIG. 1. Self-similar networks: (a) fractal dimension $D=2$ after three ($m=3$) iterations (seven branching levels), (b) construction of two branches, and (c) fractal dimension $D=1.26$ after three iterations (seven branching levels).

In order to calculate the effective thermal conductivity of the composite, we first calculate the total thermal resistance of the H-shaped fractal-like tree network shown in Fig. 1, and due to symmetry, we take the second quadrant [upper left part O-O'-O'' in Fig. 1(a)] for analysis of thermal resistance. The kth [$(k-2)$th] branching structure after m $(k=2m+1)$ iterations on an initially H-shaped structure is shown in Fig. 2(a). Figure 2(b) is the network of equivalent thermal resistances of Fig. 2(a), and Fig. 2(c) is the resistance of the network. Since $d_k << L_k$, the thermal resistance by the horizontal channel L_{k-1} is neglected for simplicity. From Fig. 2 and based on the series-parallel principle for thermal resistance, the thermal resistance $R_{t,k}$ can be found to be

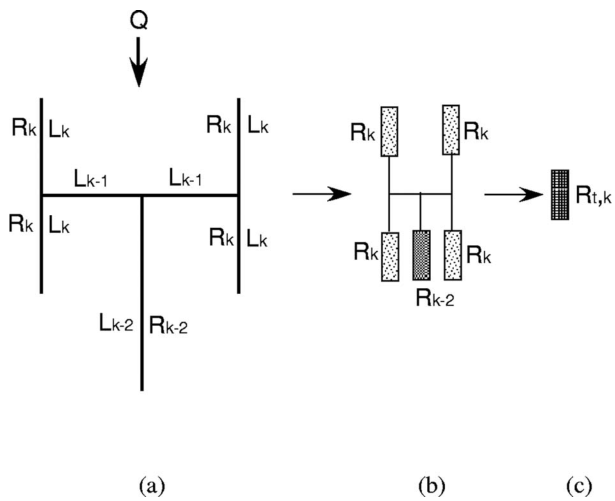

FIG. 2. (a) The kth level network, (b) the corresponding thermal resistance network, and (c) the total effective resistance of the network.

$$R_{t,k} = \frac{R_k}{2} + \frac{R_k R_{k-2}}{2R_{k-2} + R_k}. \tag{1}$$

In Eq. (1) we have neglected the resistance of the two horizontal branches L_{k-1} because the diameter of the branch L_{k-1} is assumed to be much smaller than its length. The resistance R_k in Eq. (1) can be expressed as

$$R_k = \frac{L_k}{\lambda A_k} = R_0\left(\frac{\gamma}{\beta^2}\right)^k \tag{2}$$

where $R_0 = L_0/\lambda \pi d_0^2/4 = L_0/\lambda A_0$ is the thermal resistance of the 0 level branch cylinder or channel, a single cylinder or channel, which is called an element in Bejan's work [1], and $A_0 = \pi d_0^2/4$ is the cross-section area of the 0th cylinder or channel, λ is the thermal conductivity of the branching material, L_0 is the length of the 0 level branch cylinder or channel, $\beta = d_{k+1}/d_k$, $\gamma = L_{k+1}/L_k$, and k satisfies $k=(2m+1)$, $m=1,2,3,\ldots$. The case of $m=0$ represents the initial structure, a 0 level or rank H-shaped structure. Yu and Yao [19,20] calculated the percolation and heat conduction on self-similar fractal geometries, Sierpinski-like gaskets, and Sierpinski carpets, using recursive algorithms to build the structures from the corresponding initial structures, the 0 stage, level, or rank gasket and carpet. This method was also applied to calculate the effective thermal conductivity of fractal porous media [17,18].

In Eq. (1), the cross-section area A_k of a branch is

$$A_k = \frac{\pi}{4} d_k^2 = \frac{\pi}{4} d_0^2 \beta^{2k}. \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1) yields

$$R_{t,k} = R_0\left(\frac{\gamma}{\beta^2}\right)^k \left[\frac{1}{2} + \frac{(\gamma/\beta^2)^{k-2}}{2(\gamma/\beta^2)^{k-2} + (\gamma/\beta^2)^k}\right] \tag{4}$$

Since R_0 is a constant, we therefore let $R_0 = 1$ for simplicity. Then we arrive at

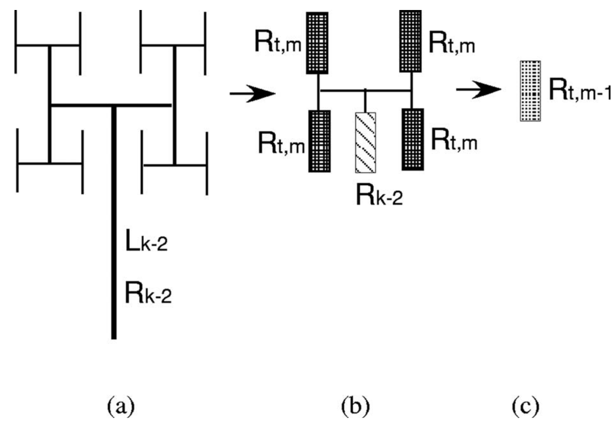

FIG. 3. (a) The (kth-1) level network, (b) the corresponding thermal resistance network, and (c) the total effective resistance of the network.

$$R_{t,k} = \left(\frac{\gamma}{\beta^2}\right)^k \left[\frac{1}{2} + \frac{(\gamma/\beta^2)^{k-2}}{2(\gamma/\beta^2)^{k-2} + (\gamma/\beta^2)^k}\right] \tag{5}$$

This equation can also be considered as the dimensionless thermal resistance of the kth level element (i.e., after the mth interaction), a basic element. We assign the resistance $R_{t,k}$ as $R_{t,m}$. Then one can calculate the total thermal resistance $R_{t,m-1}$ after the $(m-1)$th iteration shown in Fig. 3. Due to the similarity of the network, the total thermal resistance $R_{t,m-1}$ of the $(m-1)$th level is

$$\begin{aligned} R_{t,m-1} &= R_{t,m}\left(\frac{1}{2} + \frac{R_{k-2}}{2R_{k-2} + R_{t,m}}\right) \\ &= R_{t,m}\left(\frac{1}{2} + \frac{(\gamma/\beta^2)^{k-2}}{2(\gamma/\beta^2)^{k-2} + R_{t,m}}\right) \end{aligned} \tag{6}$$

where $m \geqslant 2$. It should be noted that the subscript (superscript) k in Eq. (6) is in fact less than that in Eq. (5) by 2. During iterations, instead of using the subscript $k-4$ for resistance for Fig. 3, we still use the subscript (subscript) $k-2$ for resistance, but k in Eq. (6) is reduced by 2 in each iteration. For example, if one needs to find the total resistance of a network after totally $m=4$ iterations. One should set $m=4$ and to find $k=9$, one then inserts $k=9$ into Eq. (5) to calculate $R_{t,k} = R_{t,m(=4)}$. Next, one inserts the obtained $R_{t,m}$ and $k=7$ into Eq. (6) to calculate $R_{t,m-1(=3)}$, and so on, until one obtains $R_{t,1}$ [by inserting $R_{t,2}$ and $k=3$ into the right side of Eq. (6)].

In this model, we assume that the fractal-like tree network is embedded in a two dimensional medium or matrix; thus a composite with two components (branched network and matrix material) is formed. We also assume that one dimensional heat flow passes through the composite. Therefore, the network and the matrix are in parallel in thermal, and the total effective thermal resistance of the composite can be expressed as

$$\frac{1}{R_{t,e}} = \frac{1}{R_{t,1}} + \frac{1}{R_f} \tag{7}$$

where R_f is the thermal resistance of the matrix material such as a fluid, and R_f is expressed as

$$R_f = \frac{L_e}{\lambda_f A_f} \tag{8}$$

where L_e and A_f are the effective length and cross-section area perpendicular to the heat flow and are, respectively,

$$L_e = \sum_{j=0}^{m} L_{2j+1} = L_0 \gamma \frac{1-(\gamma^2)^{m+1}}{1-\gamma^2}, \tag{9}$$

$$A_f = 2L_0 d_0. \tag{10}$$

In general, the volume of the network is small compared to the matrix, so we neglect the volume of the network material in calculation of R_f.

The total effective resistance of the composite (after m iterations of the network) can also be defined by

$$R_{t,e} = \frac{L_e}{\lambda_e A_e} \tag{11}$$

where λ_e is the effective thermal conductivity of the whole composite, and $A_e \cong A_f$ is assumed (because the volume of the network is assumed to be small compared to the matrix). The dimensionless effective thermal conductivity of the whole composite (after m iterations of the network) can be obtained from Eqs. (7) and (11) as

$$\lambda^+ = \frac{\lambda_e}{\lambda} = \frac{A_0}{L_0}\frac{L_e}{A_f}\left(\frac{1}{R_{t,1}} + \frac{\lambda_f}{\lambda}\frac{L_0}{A_0}\frac{A_f}{L_e}\right) \tag{12a}$$

$$= \frac{\lambda_f}{\lambda} + \frac{1}{R_{t,1}}\frac{A_0}{L_0}\frac{L_e}{A_f}. \tag{12b}$$

Inserting Eqs. (9) and (10) into Eq. (12) yields

$$\lambda^+ = \frac{\lambda_f}{\lambda} + \frac{1}{R_{t,1}}\frac{\pi}{8}\frac{d_0}{L_0}\gamma\frac{1-(\gamma^2)^{m+1}}{1-\gamma^2} \tag{13a}$$

$$= \lambda_1^+ + \lambda_2^+ \tag{13b}$$

where

$$\lambda_1^+ = \lambda_f/\lambda \quad \text{and } \lambda_2^+ = \frac{1}{R_{t,1}}\frac{\pi}{8}\frac{d_0}{L_0}\gamma\frac{1-(\gamma^2)^{m+1}}{1-\gamma^2}. \tag{13c}$$

In Eq. (13) $R_{t,1}$ is determined by Eq. (6). Equation (13) presents the dimensionless total effective thermal conductivity of composites with an embedded H-shaped fractal-like tree network after m iterations. Equation (13) reveals that the first term $\lambda_1^\dagger$ in Eq. (13) represents the contribution by the conductivity ratio of matrix to network material, and the second term $\lambda_2^\dagger$ can be considered as the dimensionless thermal conductivity of the network. However, the term $\lambda_2^\dagger$ is not simply the dimensionless thermal conductivity of the network. It should be noted that the resistance $R_{t,1}$ in Eq. (13) is a function of the structural parameters β, γ, and m, and $\lambda_2^\dagger$ is closely related to the structural parameters of the network.

However, if a composite has embedded in it, instead of the network, an equivalent single cylinder of length L_e [determined by Eq. (9)] and effective cross-sectional area A_e, the total effective thermal conductivity of the composite can be considered as that of the two portions (matrix and cylinder) in parallel. To calculate the total effective thermal conductivity of the composite, we need to determine the resistance of the equivalent cylinder. The cross-sectional area A_e of the equivalent cylinder can be found from

$$A_e = V/L_e \tag{14}$$

where V is the total volume of conducting material with conductivity λ of the network, and the total volume V can be found from

$$V = \sum_{j=0}^{m} 4^j \pi d_{2j+1}^2 L_{2j+1}/4 = V_0 \beta^2 \gamma \sum_{j=0}^{m} 4^j (\beta^4\gamma^2)^j \tag{15a}$$

$$= V_0 \beta^2 \gamma \frac{1-(4\beta^4\gamma^2)^{m+1}}{1-4\beta^4\gamma^2} \tag{15b}$$

where $V_0 = \pi d_0^2 L_0/4$ is the volume of a single cylinder/channel at (initial) level 0. Equation (15) indicates that if $m=0$, the total volume of conducting material with thermal conductivity λ is $V = V_0\beta^2\gamma = \pi d_1^2 L_1/4$, which is exactly the volume of a single cylinder or channel with diameter d_1 and length L_1.

Substituting Eqs. (9) and (15) into Eq. (14) yields

$$A_e = \beta^2 A_0 \frac{[1-(4\beta^4\gamma^2)^{m+1}](1-\gamma^2)}{(1-4\beta^4\gamma^2)[1-(\gamma^2)^{m+1}]}. \tag{16}$$

.

If we consider that the two portions, matrix and the cylinder with length L_e, cross-sectional area A_e, and conductivity λ, are in parallel, the total thermal resistance of the composite is

$$\frac{1}{R_{t,e}} = \frac{1}{R_f} + \frac{1}{R_e} \tag{17}$$

where R_f is determined by Eq. (8) and $R_e = L_e/(\lambda A_e)$, and here L_e and A_e are determined by Eqs. (9) and (16) respectively. After tedious calculation, the total effective dimensionless thermal conductivity of the composite is

$$\frac{\lambda_e}{\lambda} = \frac{\lambda_f}{\lambda} + \frac{\pi}{8}\frac{d_0}{L_0}\beta^2\frac{[1-(4\beta^4\gamma^2)^{m+1}](1-\gamma^2)}{(1-4\beta^4\gamma^2)[1-(\gamma^2)^{m+1}]} \tag{18a}$$

$$= \lambda_1^\dagger + \lambda_2^{\dagger\prime} \tag{18b}$$

where

$$\lambda_2^{+\prime} = \frac{\pi}{8}\frac{d_0}{L_0}\beta^2\frac{[1-(4\beta^4\gamma^2)^{m+1}](1-\gamma^2)}{(1-4\beta^4\gamma^2)[1-(\gamma^2)^{m+1}]} \tag{18c}$$

Equation (18) indicates that the dimensionless total effective dimensionless thermal conductivity of the composite is also comprised by two terms, the ratio (λ_1^+) of the component conductivities and the thermal conductivity $(\lambda_2^{\dagger\prime})$ of an equivalent cylinder with the same volume as the network. But the second term $\lambda_2^{\dagger\prime}$ in Eq. (18) is not simply the thermal conductivity of an equivalent cylinder. It is also clear that Eq. (18) is different from Eq. (13).

III. RESULTS AND DISCUSSIONS

With Eqs. (6) and (13), we can easily calculate the corresponding thermal conductivity. It should be noted that Eq. (6) requires $m \geqslant 2$. If one is only interested in the thermal conductivity of a network after only one iteration ($m=1$), i.e., three total branching levels (with $k=3$) as shown in Fig. 2, one can find the thermal resistance from Eq. (5) when the structural parameters β and γ are given and then insert $R_{t,m(=1)}$ into Eq. (13) by assuming $m=1$ to find the effective thermal conductivity of the composite.

For general cases, i.e., cases of $m \geqslant 2$, the algorithm for the dimensionless total effective thermal conductivity of composites with embedded H-shaped fractal-like tree networks is summarized as follows: (1) Given the structural parameters β, γ, m, and the ratio λ_f/λ, (2) calculate the dimensionless resistance $R_{t,k}$ after m iterations from Eq. (5) by setting $k=2m+1$, and set $R_{t,k}=R_{t,m}$ after $R_{t,k}$ is obtained. (3) calculate $R_{t,m-1}$ from Eq. (6) according to $k=2(j-l)+1$, $j=m, m-1, \ldots, 3, 2$, until $R_{t,1}$ is found and, (4) insert the obtained $R_{t,1}$ and the ratio λ_f/λ into Eq. (13a) to obtain the dimensionless total effective thermal conductivity $\lambda^{\dagger}$ of the composites. Step 3 is repeated until $m=2$.

Figure 4 presents the term $\lambda_2^{\dagger}$ in Eq. (13) versus the diameter ratio β at different m and γ obtained by the present model. The term $\lambda_2^{\dagger}$ can be considered as the dimensionless thermal conductivity primarily obtained from the contribution from the network geometric parameters. From Fig. 4 it can be seen that the term $\lambda_2^{\dagger}$ decreases with the increase of the length ratio γ. This is expected because a higher length ratio γ implies longer branches, leading to higher resistance and to lower thermal conductivity. Figure 4 also denotes that the term $\lambda_2^{\dagger}$ decreases with increase of the branching levels m. This can be explained as follows. When iterations m increase, the network becomes densely filled with much slenderer branches, leading to increase of the thermal resistance and to decrease of thermal conductivity. Figure 4 also reveals that the term $\lambda_2^{\dagger}$ decreases with decrease of the diameter ratio β and is lower than the thermal conductivity of the original material by several orders of magnitude. This means that the networks may be used as thermal insulation composities or structures. However, it should be noted that when $\beta > 0.707$, the thermal conductivity $\lambda_2^{\dagger}$ continuously increases. This phenomenon implies that no optimal diameter ratio β exists in such H-shaped fractal-like tree networks, and the effective thermal conductivity for the H-shaped fractal-like tree network does not obey Murray's law [21]. On the other hand, the case with $\beta > 0.707$ (and $\gamma > 0.707$) may also cause overlapping within limited iterations although this case might provide a network with higher thermal conductivity than that of its original material. From Fig. 4 we find that the dimensionless thermal conductivity $\lambda_2^{\dagger}$ scales with the diameter as $\lambda_1^{\dagger} \sim \beta^{4.3m}$.

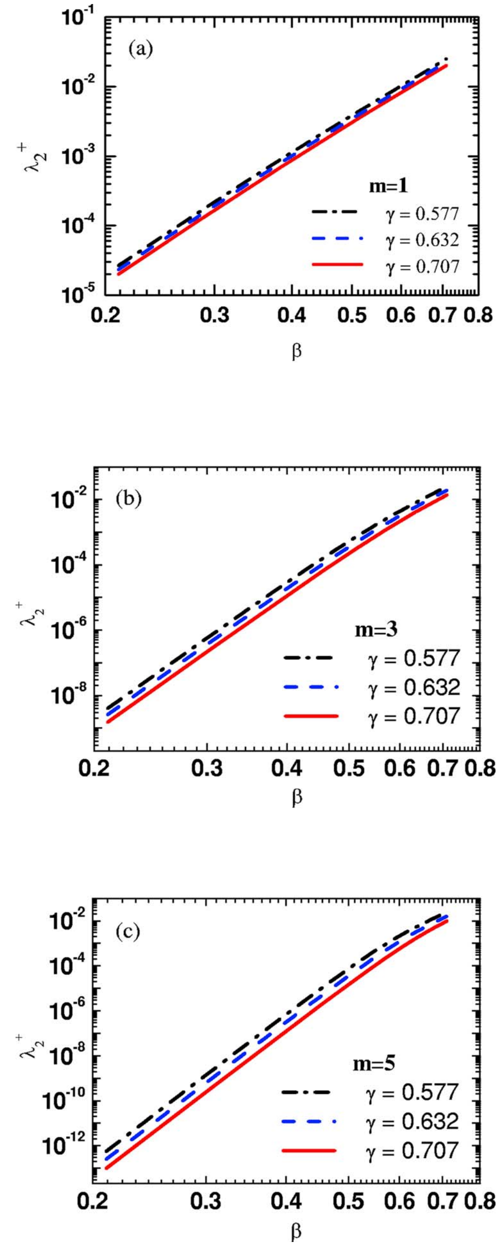

FIG. 4. (Color online) $\lambda_2^{\dagger}$ versus diameter ratios $\beta(=0.2-0.707)$ at different m and γ.

Figure 5 shows the dimensionless thermal conductivity $\lambda_2^{\dagger}$ versus iterations m at different length ratios and different diameter exponents. The results show that the dimensionless thermal conductivity $\lambda_2^{\dagger}$ decreases with increase of the iterations m, and no asymptotic value is observed as m increases. It is again interesting to see that the dimensionless thermal conductivity increases with m when $\gamma=0.5$ and $\beta=0.707$ [see Fig. 5(d)]. This can be explained as follows: $\beta=0.707$ is the maximum possible diameter ratio for preserving area and $\gamma=0.5$ is the minimum possible length ratio, leading to the minimum thermal resistance and thus to the maximum thermal conductivity.

Figure 6 shows the total effective dimensionless thermal conductivity [by Eq. (13)] of composites with embedded networks at different diameter ratios and different length ratios.

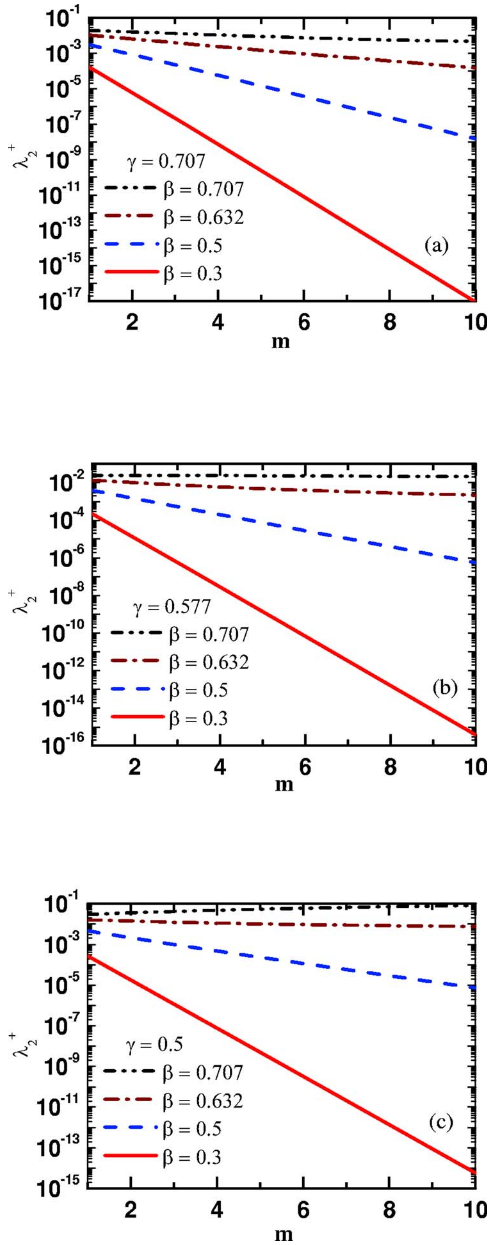

FIG. 5. (Color online) $\lambda_2^\dagger$ versus iterations m at different γ and β.

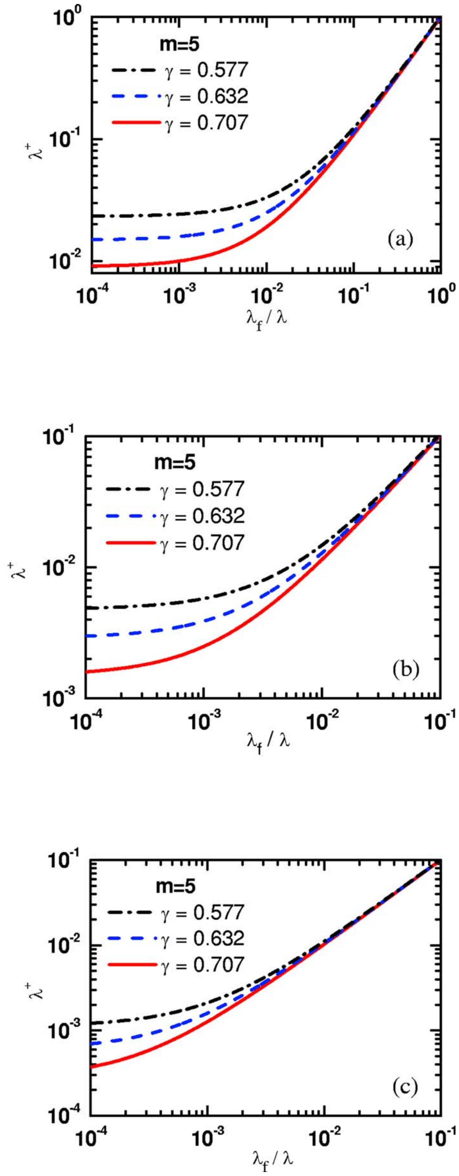

FIG. 6. (Color online) The effective thermal conductivities at β=(a) 0.707, (b) 0.632, and (c) 0.577.

It is found that the total effective dimensionless thermal conductivity is dominated by the thermal conductivity ratio of matrix material to network material, λ_f/λ, if the ratio λ_f/λ is higher than a certain value depending on the diameter ratio β. For example [see Fig. 6(a)], when β=0.707 and $\lambda_f/\lambda > 0.20$, the total effective dimensionless thermal conductivity is mainly determined by the term $\lambda_1^\dagger$ due to the lower thermal conductivity of the network. However, when $\lambda_f/\lambda < 0.20$, the total effective dimensionless thermal conductivity is mainly governed by the term $\lambda_2^\dagger$. This means that the network plays an important role when $\lambda_f/\lambda < 0.20$. The figure again shows that the total effective dimensionless thermal conductivity decreases with decrease of the diameter ratio β, this is expected.

Figure 7 compares the thermal conductivities due to the term $\lambda_2^\dagger$ in Eq. (18) with those due to the term $\lambda_2^\dagger$ in Eq. (13). It can be found that the thermal conductivity due to $\lambda_2^\dagger$ in Eq. (13) is much lower than that due to $\lambda_2^\dagger$ in Eq. (18). This means that although both cases, the fractal-like tree network and the equivalent single cylinder, have the same bulk thermal conductivity λ, the H-shaped fractal-like tree networks significantly reduce the thermal conductivity compared to the equivalent single cylinder.

Figure 8 compares the effective thermal conductivities by the different models Eqs. (13a) and (18a). The results clearly indicate that the model Eq. (18a), which is equivalent to the parallel situation of matrix and a single channel with the same volume material as the network, provides much higher [about one order of magnitude; see Fig. 8(c)] thermal conductivity than that obtained by Eq. (13a), which is equivalent to the parallel situation of the matrix and the network with

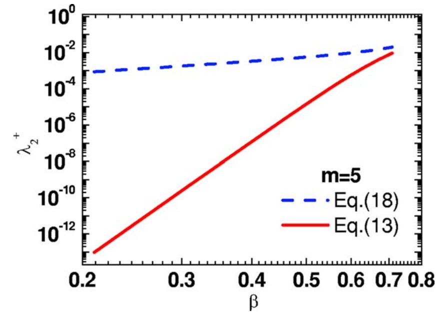

FIG. 7. (Color online) A comparison of thermal conductivities between different models.

the same volume material as a single channel. This means that a composite with embedded fractal-like tree network can provide much lower thermal conductivity as the ratio λ_f/λ is lower than a certain value such as 0.1 as shown in Fig. 8(c), depending on the values of γ and β. This implies that a composite embedded with the fractal-like tree network might be particularly useful for design of insulating devices/ equipments.

IV. CONCLUSIONS

In this paper, we have studied the effective thermal conductivity of composites with embedded H-shaped fractal-like tree networks. The effective thermal conductivity of such composites consists of two portions: the ratio of component conductivities and the portion primarily related to the networks. When the ratio of component conductivities is higher than a certain value, the thermal conductivity of the composite is mainly determined by the matrix. Otherwise, the thermal conductivities of both portions have important effects on the total thermal conductivity of the composite. It is found that the thermal conductivity of the networks is closely related to the geometry of the networks; the longer the branches, the lower the thermal conductivity; the effective thermal conductivity decreases as β decreases; the effective thermal conductivity of the networks is less than that of the original material by several orders of magnitude when the branches become slenderer and the network becomes denser (i.e., at high iterations m). It is found that the dimensionless thermal conductivity of the network scales as the diameter exponent β, $\lambda_2^{\dagger} \sim \beta^{4.3m}$. It is also found that the thermal conductivity of the H-shaped fractal-like tree networks does not obey Murray's law. The properties of the H-shaped fractal-like tree networks might have application for designing insulating or conducting structures such as space equipment. The fractal-like tree network structures can provide much lower thermal conductivity than that of an equivalent single cylinder.

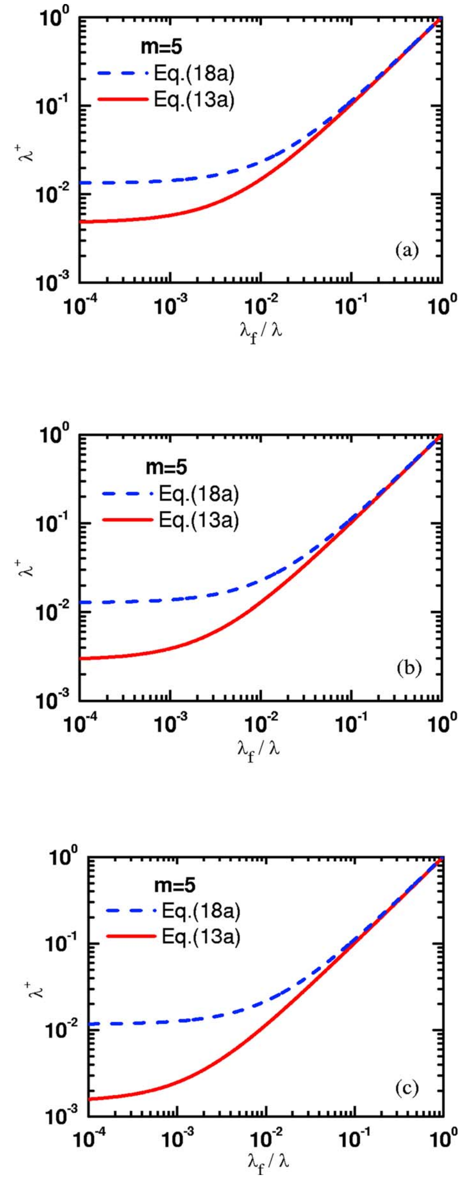

FIG. 8. (Color online) A comparison of the effective thermal conductivities based on Eqs. (13a) and (18a) at (a) γ=0.577 and β=0.632, (b) γ=0.632 and β=0.632, and (c) γ=0.707 and β =0.632.

ACKNOWLEDGMENTS

This work was supported in part by Faculty Research Grants and DSTA Singapore under Project Agreement No. DSTA POD0001821, and National Natural Science Foundation of China through Grant No. 10572052 (YU, B.).

[1] A. Bejan, Int. J. Heat Mass Transfer **40**, 799 (1997).
[2] A. Bejan, *Advanced Engineering Thermodynamics*, 2nd ed., (Wiley, New York, 1997), Chap. 13.
[3] A. Bejan, *Shape and Structure from Engineering to Nature* (Cambridge University Press, Cambridge, U.K., 2000).
[4] M. Neagu and A. Bejan, J. Appl. Phys. **86**, 1136 (1999).
[5] M. Neagu and A. Bejan, J. Appl. Phys. **86**, 7107 (1999).
[6] L. Gosselin and A. Bejan, J. Appl. Phys. **96**, 5852 (2004).
[7] M. M. Kearney, Chem. Eng. Prog. **96**, 61 (2000).
[8] Y. P. Chen and P. Cheng, Int. J. Heat Mass Transfer **45**, 2643 (2002).
[9] A. K. Silva, S. Lorente, and A. Bejan, J. Appl. Phys. **96**, 1709 (2004).
[10] D. Tondeur and L. Luo, Chem. Eng. Sci. **59**, 1799 (2004).
[11] L. Ghodoossi, Energy Convers. Manage. **46**, 771 (2005).
[12] A. Bejan, L. A. O. Rocha, and S. Lorente, Int. J. Therm. Sci. **39**, 949 (2000).
[13] B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, New York, 1982).
[14] K. Muralidhar, Int. J. Heat Mass Transfer **33**, 1759 (1990).
[15] C. T. Hus, P. Cheng, and K. W. Wong, J. Heat Transfer **117**, 264 (1995).
[16] B. M. Yu and P. Cheng, J. Thermophys. Heat Transfer **16**, 22 (2002).
[17] Y. T. Ma, B. M. Yu, D. M. Zhang, and M. Q. Zou, J. Appl. Phys. **95**, 6426 (2004).
[18] Y. J. Feng, B. M. Yu, M. Q. Zou, and D. M. Zhang, J. Phys. D **37**, 3030 (2004).
[19] B. M. Yu and K. L. Yao, J. Phys. A **21**, 3269 (1988).
[20] B. M. Yu and K. L. Yao, Phys. Rev. A **44**, 3664 (1991).
[21] C. D. Murray, Proc. Natl. Acad. Sci. U.S.A. **12**, 207 (1926).